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Determination of permeability using electrical properties of reservoir rocks by the critical path analysis

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ABSTRACT:Description methods of reservoir parameters are valuable and the relationship between permeability and electrical properties have been always faced to difficulties due to the completing of the carbonate rocks pore space. Therefore, in this article we try to study the relationship between permeability and electrical properties of the reservoir rocks.In the first stage of this research is theoretically studied the relationship between the ratio permeability to electrical conductivity and hydraulic conductivity based to the pore structure, the coordination number, and the shape and pore radius of the sandstone samples. Then by using the literature review, the pore network model is presented that requires critical path analysis (CPA) with applying the experimental data. The proposed model with the pore radius values and the different number of coordination can predict the values for the ratio permeability to electrical and hydraulic conductivity. The results of CPA method have been compared with experimental data. These results are showed an acceptable fit between the model and experimental data, and also indicated that the sandstone samples have the lowest error and predicted the best values for the coordination number three.
Keywords: permeability, electrical conductivity, critical path analysis, sandstone

INTRODUCTION

Predict the macroscopic transitive properties of porous substance is one of the prolonged problems that has numerous applications in the fields of hydrology, petroleum engineering, and chemical engineering. It requires some information such as permeability and electrical conductivity of macroscopic transitive coefficients in the pores [1]. Macroscopic parameters could not theoretically be considered. In the previous theoretical studies, fluid and transitive properties in porous network are simulated by area - pore network method [2]. This method is based on the photography from tortuosity of the pore space network with order or disorder bonds. To extract a geometric model usually assumes that the all pores are related through the pore throats. Network models will be used for simulation a variety of transitions in porous rocks if the shape, size, relations and spatial arrangement of pores are adequately identified. Many researchers have used permeable theory for seeking to understand of micro scale in porous zone [3], [4]. This theory is used for calculating the transition macroscopic coefficients in heterogeneous zone. Todd and Skaggs stated the following equation by using permeable theory and critical path analysis [5]:

$$\frac{k}{EC_b/EC_w} = C_{KT}\delta_C^2 \qquad (1)$$

The history of the hydraulic conductivity theory comes back to Bernabe and Bruderer [6]. The following equation expresses the relation that hydraulic conductivity has in the pore networks:

$$K = \frac{r^2}{cF} \qquad (2)$$

Kozeny and Carman have shown r value for a cylindrical shape as following equation [7,8]:

$$r = 2\frac{<V_i>}{<A_i>} \qquad (3)$$

[9] Relationship between hydraulic conductivity and electrical with different length scales of r for K is expressed as follows:

$$\Lambda = 2\frac{\int E^2 dv}{\int E^2 dA} \qquad (4)$$

Koplik in 1984, examined the behavior of fluid in the pores [10].

$$Q_{ij} \propto \frac{r^4 p_t}{l_0} \qquad (5)$$

Katz and Thompson obtained the critical path analysis behavior by helping the research of Ambegaokar and his colleague [11, 12].

$$K = \frac{r_c^2}{cF} \quad (6)$$

There is a difference between the relations (2) and (6). In the (2) equation c = 8, but in (6) equation c= 5.56 and r_c is critical pore radius.

Critical path analysis is applicable to find r_c . For determining the hydraulic conductivity in addition r_c to have the value of the pore critical length is necessary. Friedman and Seaton presented two relations hydraulic conductivity and electrical by helping the analysis of the relationship between stated using critical path [13]:

$$g_e^h = \frac{\pi}{8\mu l_0} r_c^4 \quad (7)$$

And electrical conductivity

$$g_e^e = EC \frac{\pi r_c^2}{l_0} \quad (8)$$

METHODOLOGY

Rocks are not quite filled and always contain pores and cavities in the case of related or not related. These pores have a great importance in the rocks mechanical properties. Generally, when rock porosity is greater, mechanical resistance is lower. If the holes and cracks of stone are formed a connected network, in this case the gap is greater permeability is higher. But, if the pores are separated, they cannot represent the rock permeability. Pores conductive can be expressed as follows for saturated and order pores network with fixed-length l_0:

$$g = g_0 \delta^m \quad (9)$$

Where δ is characteristic pore length, g_0 and m are fixed values. g_0 and m are depended to the geometric shape, intrinsic transition and fluid viscosity or electrical conductivity. Thus g may be related to the electrical conductivity and hydraulic conductivity. Also δ is defined as the pore diameter for cylindrical shape pores where as it is the pore width for slit shape pores[14].

Electrical and hydraulic conductivity of pore networks

K is hydraulic conductivity and proportionality constant between water flow (special db) and hydraulic gradient in Darcy's law. If an incompressible fluid transit in a horizontal and linear way from a rock sample with a length of L and cross sectional area of A, it follows from the Darcy equation:

$$V = -\frac{K}{\mu}\frac{dp}{dL}, \quad V = \frac{q}{A} \quad (10)$$

$$q = -\frac{KA}{\mu}\frac{dp}{dL} \quad (11)$$

$$q = -K\nabla h \quad (12)$$

$$K = k\left(\frac{\rho g}{\mu}\right) \quad (13)$$

Since in this study the effect of pore structure on hydraulic conductivity is examined; therefor, permeability k is paid more attention.

Similarly, we have only studied the ratio between electrical conductivity appearance EC_a and hydraulic conductivity of the saturated EC_w , the other words $F = {EC_a}/{EC_w}$ (formation factor) which is determined by the pore structure. EC_a and EC_w are amount the proportionality constant between the electric current per unit area $\frac{i}{A}$ and the gradient of the electric potential V based on Ohm's law [15]:

$$I = \frac{V}{R}$$

$$\frac{i}{A} = (EC)\nabla V \quad (14)$$

To determine the pore structure can be considered coordination number.

Coordination Number

The number of ions of opposite charge that there are around a specific ion is said coordination number. Coordination number an ionic crystal solid depends on the relative sizes of the cations and anions. Arrangement of the ions in the crystal of an ionic compound in depending on the size of the anion and cation follow a specific pattern. This pattern is repeated throughout the crystal, for example, the sodium chloride has coordination number 6 and cesium chloride has coordination number 8. Therefore, the shape of the unit cell and the arranged ions next to each other depend on the size of the ions. Number coordination of each combination identify the its structure. This property has been used in the study for determining the connection pores and pore size in pore three-dimensional network. While in this study, ions are considered as pores in pore three-dimensional network. Coordination number for simple cubic lattice is considered 6 = z. Simple cubic network topology can be different

from the dissolution process by randomly eliminating some of the bands. As a result a network with $z \leq 6$, with some bands that are completely isolated or those jointed with different bands are determined [13,17,18].
Electric current in a cylindrical pore j_i^e found that obeys Ohm's law:

$$j_i^e = g^e(r)\Delta V_i \quad (15)$$

g^e Electrical conductivity and ΔV_i electrical potential difference across the length of the pores.

$$I = gV \quad (16)$$

Electrical current of the saturated solution with EC_w related:

$$g^e(r) = \frac{\pi r^2 EC_w}{l} \quad (17)$$

In the same manner the water discharge j_i^h and g^h the hydraulic conductivity of a pore is related with pressure drop ΔP_i . Equation Poiseuille for cylindrical shape pores:

$$j_i^h = g^h(r)\Delta P_i \quad (18)$$

$$g^h(r) = \frac{\pi r^4}{8\mu l} \quad (19)$$

There is also another form of pores with slit shape as follows:

$$g^e(w) = \frac{bw}{l} EC_w \quad (20)$$

$$g^h(w) = \frac{bw^3}{12\mu l} \quad (21)$$

Different mechanisms of displacement is determined by pore conductivity in size. For cylindrical pores $g^e \propto r^2$ and $g^h \propto r^4$ and for the pore slit shaped pore, and $g^e \propto w$ is $g^h \propto w^3$ [19].
In this study, is only examined the cylindrical shaped pore. We used the log normal distribution method, because of the heterogeneity of the pore radius.

Log Normal Distribution

Log-normal distribution is as follows:

$$F_g(\delta) = \frac{1}{\sqrt{2\pi}\sigma\delta} exp\left[-\left(\frac{Ln\delta-\mu}{\sqrt{2}\sigma}\right)^2\right] \quad (22)$$

Population mean μ and standard deviation σ. In this study, the accuracy of the log-normal function for the experimental data is determined by using the fit goodness. After approving the function is calculated the critical pore radius.
Fit goodness is a statistical method to assess the accuracy. There are different tests in this method that we used Kolmogorov-Smirnov test. The p-value is calculated by applying the statistical software R. The values of μ and σ is determined by the method of MLE (Maximum Likelihood Estimate) that the estimation method is used:

$$\mu\hat{}_{ML} = Ln(x) \quad (23)$$

$$\sigma\hat{}_{ML} = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(d_i - d)^2} \quad (24)$$

Where n is the number of data and d = Ln (x) .
The fit of data is presented in Fig 1 in the form of histograms plotted for the entire sample. The pore size distribution is defined in terms of a probability density function. Pore radius distribution function is shown by the following equation:

$$f(r) = \frac{1}{\sqrt{2\pi}\sigma r} exp\left[-\left(\frac{Lnr-\mu}{\sqrt{2}\sigma}\right)^2\right] \quad (25)$$

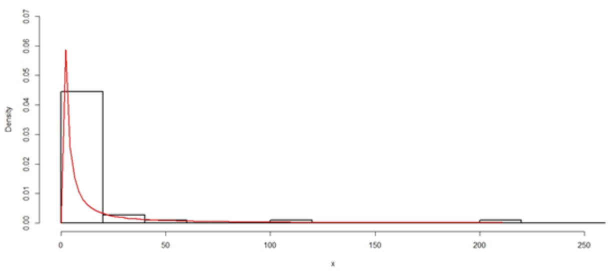

Figure 1. goodness of fit the pore size distribution with a log-normal distribution

Critical Path Analysis (CPA)

(CPA) is one of the methods for analyzing the network. A network is a picture of project that shows the project activities and the relationships between them. There is at least one the longest path in the project network that is called the critical path. This method may be used in the most probable location. Critical path analysis can be used in systems that are heterogeneous. The critical radius is the radius of connections or links that may be determined from one side to the other side of network. All models critical path analysis are focused on conductivity that depends on throat and critical value. Path analysis states that which way is more important [12]. CPA can be used to find the critical radius r_{crit} (for simplicity r_c) to predict the relationship between electrical conductivity and permeability and hydraulic conductivity. CPA objective is analyzing the paths that are crossing on throat to show the likelihood of permeability, the electrical and hydraulic conductivity.

Three-dimensional pore network

To extract a geometric model is usually assumed that the pore space is a set of interconnected pores through local constraints. Local constraints are as pore throats. Accordingly, the pore network model has been made by the pore size and pore throats in order to model the spatial distribution and the network connection. Simulated model are assumed according to the cylindrical shaped pores (eg, porous spherical and cylindrical throat) to predict the behavior of macroscopic for the microscopic structure. It requires to topological and geometric information, such as pore size distribution, throat, coordination number, pore distribution and correlation of pore. By using the critical path analysis is studied the relationship between two different transmission properties in three-dimensional pore network. Two transmission properties is considered in this study are: Hydraulic conductivity and electrical conductivity.

Three-dimensional pore network by a simple cubic lattice with different coordination numbers and log-normal distribution are shown.

Hydraulic and electrical properties

The ratio of electrical conductivity to hydraulic conductivity is applied to remove many of the constants depends on network. The relationship between the displacement properties are characterized by r_{crit}. K-EC concerning the relations (17) and (19) as follows:

$$\frac{g_{crit}^{h}}{g_{crit}^{e}} = \frac{r_{crit}^{2}}{8\mu EC_w} \quad (26)$$

$$k = \frac{{r_{crit}}^2}{8}\frac{EC_a}{EC_w} \quad (27)$$

Critical path analysis approximate for ratio between hydraulic conductivity and electrical conductivity is proportional to ${r_{crit}}^2$. The use of critical path analysis model for hydraulic and electrical conductivity of the details are not required, only the critical pore size and pore geometry are important. Viscosity and permeability can be normalized by k_m ($k_m = \frac{m^2}{8}$), where m is the mean pore radius. Thise equation can be written as follows:

$$\frac{k/k_m}{EC_a/EC_w} = ({r_{crit}}/{m})^2 \quad (28)$$

Calculate the critical radius (r_{crit})

The simplest model for analyzing the critical path of a network of flow in porous media under saturated conditions. Each of the links by the throat and random radius from distribution $f(r)$have been selected to r_{crit} the following equation is calculated:

$$p_c = \int_{r_{crit}}^{\infty} f(r)dr \quad (29)$$

Where $f(r)$ log-normal distribution function and probability p_c link (likely permeability) shows. [20]

On the other hand, p_c depends on the dimension of the network and the network structure that it is calculated with different coordination number is calculated, this relationship is shown as follows:

$$zp_c = \frac{d}{d-1} \quad (30)$$

d is dimension.

As a result, we have a three-dimensional network:

$$\frac{1.5}{z} = \int_{r_{crit}}^{\infty} f(r)dr \quad (31)$$

The relation (31) to determine the critical radius for coordination numbers 2, 3, 4, 5 and 6 are obtained using the normal distribution table.

ANALYSIS OF RESULTS

The critical path analysis is applied to calculate the critical radius for each coordination r_{crit} z (2≤z≤6). The results in Table 1 are presented. The ratio $\frac{k}{k_m}$ to $^{EC_a}/_{EC_w}$ is predicted by helping equation (27) and different coordination numbers. That the results are shown in Table 2. In Table 3 the predicted values are compared with experimental data. Finally, in Figure 2, the experimental data with the modeled values for different coordination numbers are drawn.

Table 1. the calculated critical radius for 21 sample with different coordination number (6> z> 2)

r_{crit}(μm) z = 6	z = 5	z = 4	z = 3	z = 2
11.023	8.011	4.610	1.765	1.319
11.011	7.994	4.598	1.759	1.315
11.134	8.082	4.647	1.777	1.328
14.673	10.950	6.605	2.745	2.104
11.026	8.005	4.603	1.761	1.316
9.747	7.125	4.147	1.620	1.218
11.175	8.116	4.671	1.789	1.337
11.162	8.104	4.661	1.783	1.333
14.464	10.364	5.827	2.143	1.583
11.149	8.094	4.656	1.781	1.331
11.018	8.000	4.602	1.761	1.316
11.012	7.995	4.598	1.759	1.315
11.163	8.104	4.660	1.783	1.332
11.017	7.999	4.602	1.762	1.317
11.161	8.102	4.660	1.782	1.332
9.434	6.911	4.037	1.587	1.196
9.374	6.864	4.007	1.573	1.185
11.776	8.642	5.064	2.001	1.511
11.150	8.097	4.660	1.785	1.334
11.121	8.073	4.642	1.775	1.327
11.014	7.995	4.598	1.759	1.314

Table 2. the ratio of the predicted permeability and hydraulic conductivity and electrical coordination and the actual ratio from experimental data

		r_{crit}^2/m^2			$\frac{k/k_m}{EC_a/EC_w}$
z = 6	z = 5	z = 4	z = 3	z = 2	
0.175	0.210	0.288	0.499	0.823	0.428
0.083	0.106	0.161	0.333	0.644	0.300
0.088	0.112	0.166	0.333	0.625	0.383
0.088	0.112	0.166	0.333	0.625	0.429
0.088	0.112	0.166	0.333	0.625	0.129
0.088	0.112	0.166	0.333	0.625	0.012
0.088	0.112	0.166	0.333	0.625	0.347
0.088	0.112	0.166	0.333	0.625	0.624
0.175	0.210	0.288	0.499	0.823	0.448
0.175	0.210	0.288	0.499	0.823	0.543
0.175	0.210	0.288	0.499	0.823	0.312
0.174	0.209	0.286	0.496	0.818	0.483
0.175	0.210	0.288	0.500	0.823	0.661
0.175	0.210	0.288	0.499	0.823	0.295
0.175	0.210	0.288	0.499	0.823	0.199
0.175	0.210	0.288	0.499	0.823	0.175
0.175	0.210	0.288	0.499	0.823	0.494
0.175	0.210	0.288	0.500	0.823	0.464
0.175	0.210	0.288	0.499	0.823	0.175
0.175	0.210	0.288	0.500	0.823	0.711
0.175	0.210	0.288	0.499	0.823	0.327
8%	7%	4%	3%	17%	eror%

The diagram predicted values for coordination numbers 2,3,4,5,6 1,2,3,4,5 series specified in the order for the Series 6 is a diagram based on experimental data. From comparison of the predicted values of k-EC with the actual values of k-EC can be concluded that the predicted values in coordination number with z= 3 has the better fit than the other coordination numbers. Because the laboratory samples were taken from the sandstone and coordination number attributed to sandstone is number 3.

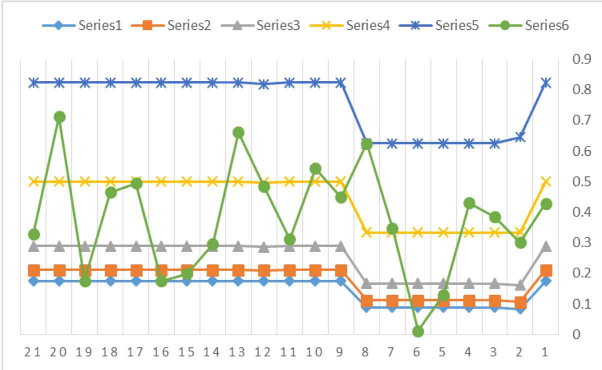

Figure 2. green curve (series 6) based on experimental data and other curves based on modeling are a number of different coordination.

CONCLUSION

The critical path analysis with ratio of k-EC shows a good response for coordination number 3. Reponses (CPA) to predict ratio K-EC just with having pore radius determines a good result for structure of the rock lithology. In critical path analysis for sandstone samples, the maximum error is in the coordination number 2.

Since samples were sandstone and z =3 and the lowest error is 3%; therefore, z = 3 can be attributed to the sandstone. The most appropriate predict could be based on different coordination numbers for ratio the permeability to the electrical conductivity.

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Notation

b	breadth of slit-shaped pore
g0	proportionality constant in the pore conductance model
K	hydraulic conductivity
rc	critical value of pore throats
Z	coordination number
μ	fuid viscosity
Λ	length scale
E	electric field
G	hydraulic conductance
ge	electrical conductance
f(r)	probability density function for pore radius [L-1].
ECa	apparent electrical conductivity of the porous medium [Q2T/ML3]
ECw	electrical conductivity of the free solution saturating the porous medium [Q2T/ML3]
k	viscous permeability [L2]
k_m	viscous permeability of a network with equal pores of the mean radius, divided by n/3, $k_m = m^2/8$ [L^2]
	pore length [L]
P	pressure [M/LT^2]
P_c	percolation threshold
r	pore radius [L]
V	electrical potential [ML^2/QT^2]
m	dynamic viscosity of the fluid [M/LT]
w	width of the slit-shaped pore [L]
crit	critical pore radius/conductance according to the CPA
δ_C	critical pore diameter
<>	represents the average volume and pore size